however, one has to be impressed by the width of the scope of the study and the very considerable mathematical culture of academician Mikhlin. This makes for quite pleasant reading.

V. T.

2[65–01, 65N30].—Barna Szabó & Ivo Babuška, *Finite Element Analysis*, Wiley, New York, 1991, xv +368 pp., 24 cm. Price $59.95.

Babuška and Szabó have pioneered the use of high-order polynomials as an alternative to mesh refinement in the finite element method. That this is a significant part of the subject of the book is therefore no surprise. However, given the different primary disciplines represented by the two authors, one could not predict a priori whether such a book would take a mathematical or an engineering approach, or perhaps some novel combination of the two. The order of authorship hints that an engineering approach is to be emphasized, and this is reflected in the "matrix method" notation that is used, but the use of mathematical ideas is also an essential part of the text.

The goals of the book are best stated by the authors themselves in the Preface.

> Our purpose in writing this book is to introduce the finite element method to engineers and engineering students in the context of the engineering decision-making process. Basic engineering and mathematical concepts are the starting points. Key theoretical results are summarized and illustrated by examples. Focus is on the developments in finite element analysis technology during the 1980s and their impact on reliability, quality assurance procedures in finite element computations, and performance. The principles that guide the construction of mathematical models are described and illustrated by examples.

Numerous books on the finite element method with a variety of objectives have appeared recently, so many that it would be quite lengthy to compare even a representative number of them. However, we will venture one comparison with a mathematical audience in mind. The present book has extensive detail with regard to examples, and its coverage of topics in linear elasticity is exhaustive. Both of these are essential for the book to be successful with an engineering audience. The book by Claes Johnson [2], on the other hand, offers a more conventionally mathematical approach to the subject. That book covers a more diverse set of topics than do Babuška and Szabó, but it lacks details of computer implementation and omits many practical issues covered by them.

In addition to providing an up-to-date survey of high-order-polynomial methods, the book includes other material not previously available in textbook form. The book presents one chapter on efficient computation of stresses using post-processing techniques to enhance the quality of the solution, and another on error estimation and control.

There is a chapter on "Miscellaneous Topics" that gives some hints about areas for further study. Included are a discussion of mixed methods and a sample nonlinear problem. However, there are some standard topics not covered in the book that may be of interest to students, such as that of viscous fluid flow

[1] (although there is one chapter on potential flow) and nonlinear models in solid mechanics, such as plasticity.

L. R. S.

1. Max D. Gunzburger, *Finite element methods for viscous incompressible flows*, Academic Press, Boston, 1989. [Review **40**, Math. Comp. **57** (1991), 871–873.]

2. Claes Johnson, *Numerical solutions of partial differential equations by the finite element method*, Cambridge Univ. Press, 1987. [Review **1**, Math. Comp. **52** (1989), 247–249.]

3[65Cxx, 65Mxx, 65Nxx].—W. E. Schiesser, *The Numerical Method of Lines: Integration of Partial Differential Equations*, Academic Press, San Diego, 1991, xiii + 326 pp., $23\frac{1}{2}$ cm. Price $69.95.

The numerical method of lines for time-dependent PDEs consists of forming a spatially discrete system of ordinary differential (or possibly differential-algebraic) equations in time and then calling a suitable ODE or DAE integrator. The method is quite powerful and versatile, and is widely used in conjunction with the wide array of high-quality ODE and DAE solvers now available.

This book presents, mainly by example, one approach, or paradigm, for the numerical method of lines (NUMOL). It fills in the details of the method by establishing a complete Fortran program template in which the discrete representation of spatial operators is done by finite differences in a certain set of Fortran software (the DSS routines), and the coding for problem specification and I/O is to be inserted into a few specific spots, with heavy use of certain COMMON structures. Relegating the discretization to a set of black boxes and imposing a structure on the problem-specific coding makes it relatively straightforward to set up and solve a problem. But the price for this is a certain loss of flexibility in the areas of spatial discretization, boundary condition representation, and treatment of PDE systems.

Aside from certain limitations, and some minor errors and omissions, the book provides a very good introduction to the numerical method of lines. It assumes very little in the way of technical background of the reader. For example, the concepts of a PDE as a physical model, of Taylor series approximation, and of eigenvalues of a matrix, are all introduced from scratch as needed. Any scientist or engineer with a little Fortran background can read this book and quickly learn to solve some interesting problems. As an introduction to the subject, the book does not treat some of the more advanced aspects, such as mixed derivatives, irregular regions, or outflow boundary conditions, and it only briefly touches on the issues of nonuniform grid selection, differential-algebraic system problems, and the efficient treatment of large stiff systems. Fans of finite-element-type versions of the method of lines will not be accommodated by this book.

Chapter 1 does a good job of introducing the basic ideas of NUMOL, with the heat equation as the (much repeated) example. The various steps are explained in great detail, although some features of the procedure could use even more discussion.

A disturbing practice that appears in Chapter 1, and is continued later, is that of forming the discrete second derivative by two successive applications of a (central) first-derivative approximation ("stagewise"), rather than by direct